Networks of integrate-and-fire neuron using rank order coding A: How to implement spike time dependent Hebbian plasticity

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Abstract

Based on neurophysiological observations on the behavior of synapses, spike time dependent Hebbian plasticity is a novel extension to the modeling of the Hebb rule. This rule has enormous importance in the learning of spiking neural networks (SNN) but its mechanisms and computational properties are still to be explored.

In this article, we present a generative model for spike time dependent plasticity based on a simplified model of the synaptic kinetic. We then explore the fitting of this model to experimental data and review some of its dynamical properties. Finally, we extend this model to a simplified model of integrate-and-fire (IF) neurons network using rank order coding. © 2001 Elsevier Science B.V. All rights reserved.

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1. The Hebb rule in neuronal modeling and SNNs

In the context of studying the adaptability of an image processing system of a robotic vehicle using a neuromimetic spiking neural network, we faced the problem of finding an unsupervised learning rule detecting the statistical features of the input. Due to its simplicity and biological plausibility, we have chosen to implement an Hebbian-like learning based on the temporal relation between pre- and post-synaptic firing.

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Let us remember Hebb's formulation:

> When an axon of cell *A* is near enough to excite a cell *B* and repeatedly or persistently takes part in firing it, some growth process or metabolic change takes place in one or both cells such that *A*'s efficiency, as one of the cells firing *B*, is increased [5].

This rule may be interpreted as a strengthening of the wiring between the neurons that form a causal chain. We may reformulate it in the context of SNNs as a temporal Hebb rule: 'When there exist synapses between two neurons, the ones transmitting presynaptic spikes until the emission of the postsynaptic spike are reinforced, the others weakened'.

To model it, we first used a binary rule that reinforced (resp. inhibited) equally the synapses activated before (resp. after) the postsynaptic spike. Actually, biological learning mechanisms depend on very small difference in the latencies between the input and the output cell discharge date [6,1].

In these last experiences, repeated pairing of a pre- and a post-synaptic spike was applied to a synapse leading to potentiation and depression similar to the temporal Hebb rule. But more precisely, the amplitude of the relative change in the weights was exponentially decreasing with the absolute spike time difference on a time scale of ∼20 ms. We present here a general synaptic model for spike time dependent plasticity.

2. A generative model for spike time dependant plasticity

2.1. Description of the model

The kinetic models of synaptic transmission are succesful descriptions of the behavior of the concentration of the different ions that lead to potential difference across the neuron's membrane and therefore, to neuronal information processing.

In our model, we will consider that neurons are emitting pulses (or spikes) that propagate to the axon but also back to the dendrites; the synapses are the contact sites between them. Synapses are considered like filters transmitting a current to the postsynaptic site, it is linear under the assumption of low spike frequencies: the response to a spike is its impulse response or post synaptic pulse (PSP). The synapses may be valuated by the amplitude of the PSP and are, therefore, characterized by a weight $g \cdot g_{max}$, where g_{max} is the maximum possible weight and g ranges from 0 to 1.

On one side, we will consider a pool of emitters (corresponding to the docked vesicles containing neurotransmitter) quantified by their relative concentration C. This quantity is triggered by presynaptic spikes but this pool is limited (leading to synaptic depression). On the other side, we will consider a pool of receivers (corresponding to the sensitivity of postsynaptic sites and that appear to be calcium related mechanism) similarly quantified by D. This quantity is mediated by postsynaptic spikes and may be related to synaptic facilitation in our model.

2.2. Modeling the synapses' dynamics by kinetic equations

Modeling C (resp. D) for the synapse (A–B) with a first order kinetic pulse based model of decay time constant τ_C and pulse amplitude α_C (resp., τ_D and α_D), we get by noting as in [4] the presynaptic spike times by t_k^a (resp., postsynaptic by t_k^{out}):

$$\frac{\mathrm{d}C^{(A-B)}}{\mathrm{d}t} = -\frac{1}{\tau_C}C^{(A-B)} + \alpha_C\sum_k \delta(t - t_k^{(A-B)})(1 - C^{(A-B)}),$$

$$\frac{\mathrm{d}D}{\mathrm{d}t} = -\frac{1}{\tau_D}D + \alpha_D\sum_j \delta(t - t_j^{\text{out}})(1 - D).$$

Finally, we may model the variation of g according to our definition of synaptic weight like the combination of an excitation relative to the synapse strength and proportional to the emitters' concentration when a postsynaptic spike arrives and an inhibition relative to the existing synapse weakness and proportional to receivers' concentration when a presynaptic spike arrives. Modeling it with a first order kinetic of decay time constant τ_g, we get

$$\tau_g\frac{\mathrm{d}g^{(A-B)}}{\mathrm{d}t} = +(1 - g^{(A-B)})C\sum_j \delta(t - t_j^{\text{out}}) - g^{(A-B)}D\sum_k \delta(t - t_k^{(A-B)}).$$

This formulation has the computational advantage of depending only on the present state in comparison with rules that need to store spike timings in a certain window.

3. Results

3.1. Fitting neurophysiological results

Replicating the conditions of [6] we may easily obtain for a given initial weight an analytical formulation for the relative change for our model. The constant of our model (the time constants and amplitudes of C and D) correspond to the time constant and amplitude of the explicit rule, respectively, before and after the postsynaptic spike (see Fig. 1A).

We also find that the resulting explicit expression of weight change according to the initial weight (see Fig. 1B) is analogous to the multiplicative rule described in [1] and analytically studied in [7].

3.2. Rule's properties

The rule we propose is, therefore, a natural extension of the explicit rules described in [7,8]. We studied it using different models of neurons like integrate-and-fire [2] or conductance based IF [3], resulting in similar results as in [7,8]:

- *Adaptation to random input*: Given a random input (like a spike train), the neuron tends to generate a spike train with a great variability in the interspike interval (ISI) distribution.
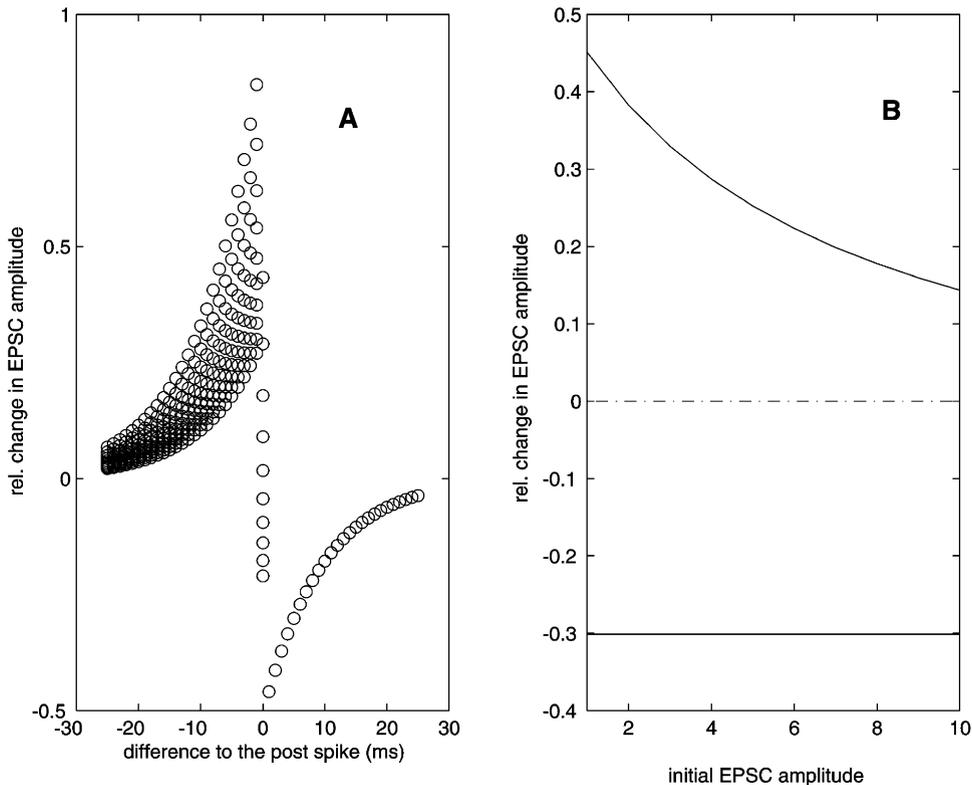
Fig. 1. Simulation of the synapse model. (A) Relative weight variation for different initial weights. The conditions of the simulation are replicated from [7]. The weights are strengthened if the presynaptic spike is coming before the postsynaptic spike. The amplitude of change decreases exponentially with the time difference. (B) Relative weight variation depending on the initial weight (replicating the conditions of [2] i.e. a time difference of $\pm$ 6 ms. The rule is multiplicative: decreasing with weight amplitude for potentiation; constant for depotentiation.

- *Learning to detect a synchronized input*: The rule is adapted to cluster synapses that receive synfire inputs, leading to a bimodal weight distribution differentiating synchronized and unsynchronized inputs.
- *Asynchronous wave detection*: Under certain conditions, this rule may provide a way to detect temporal patterns.

3.3. Application to rank order coding (ROC)

Neuropsychological experiments have recently shown the incompatibility of the classical rate code introduced in 1926 by Adrian and Zotterman. Actually, the speed of processing in the visual system can be as short as 150 ms for a complex task involving at least 10 processing layers [9] leaving less than 10 ms for the neuron to process and
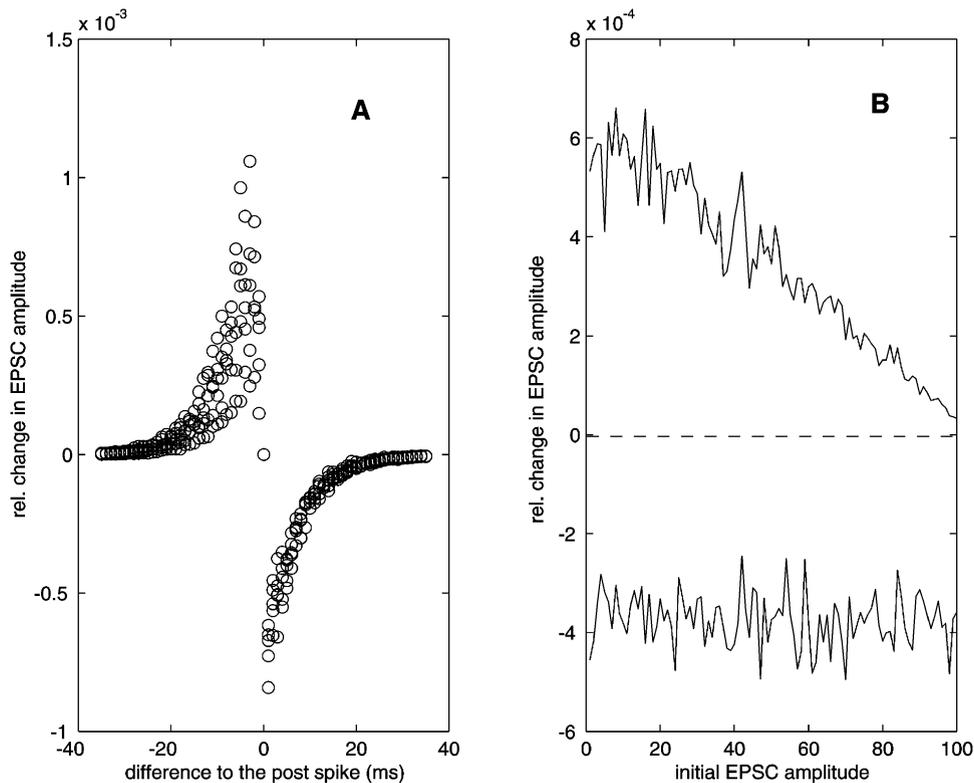
Fig. 2. Simulation of the synapse's model based on rank order coding. (A) Relative weight variation. As an impulse wave (100 synapses, one spike max per neuron, timing according to a modified Poisson distribution), arrives at the synapses, the relative weight change based on a rank order rule is similar to Fig. 1. (B) Relative weight variation depending on the initial weight, we still have a multiplicative rule.

transmit information. A solution is to use the asynchronous characteristics of the neural code, as in the analog-to-delay conversion characteristic of retinal ganglion cells by using integrate-and-fire like models of neurons: generally, the stronger the input intensity, the quicker the neuron would fire.

We, therefore, propose a model using only one spike per neuron, and simplifying the IF's dynamic, we study a network of neurons using the rank of the incoming spikes as a possible coding. Similarly, as for the first synaptic model, we may study a rule that relies on the order of the presynaptic spikes relative to the postsynaptic firing with a similar shape (or kinetic rules). By simulating this rule on a temporal scale but with the rule based on ROC, with input distributed as a biologically adapted Poisson distribution, we obtain similar results as in Fig. 1 (see Fig. 2). Further, results are described in the companion paper (same volume): A. Delorme et al. *Networks of Integrate-and-Fire Neurons using Rank Order Coding B*: *Spike Timing Dependent Plasticity and Emergence of Orientation Selectivity.*

To conclude, faced with unsatisfactory model of the mechanisms underlying spike time dependent plasticity [8], we found a generative model that is compatible with electrophysiological results and is able to explain the relation between synapse strength and percentage of change in synaptic weight [1]. We also showed that such a rule can comply with different coding scheme that include latency coding and rank order coding.

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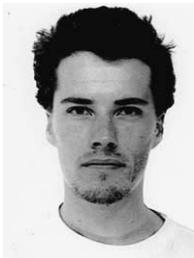
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